vibrazioni

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11 novembre 2025

piccole oscillazioni

$$V = V_{x_e} + \left(\frac{\partial V}{\partial x}\right)_{x_e}(x - x_e) + \frac{1}{2}\left(\frac{\partial^2 V}{\partial x^2}\right)_{x_e}(x - x_e)^2 + \dots$$

meccanica classica

$$m_1 x_1'' = -k(x_1 - x_2)$$
$$m_2 x_2'' = -k(x_2 - x_1)$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}'' = \begin{bmatrix} x_1'' \\ x_2'' \end{bmatrix} = \begin{bmatrix} -\frac{k}{m_1} & \frac{k}{m_1} \\ \frac{k}{m_2} & -\frac{k}{m_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{1}$$

disaccoppiamento

so risolvere

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}'' = - \begin{bmatrix} \omega_1^2 & 0 \\ 0 & \omega_2^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{2}$$

$$\begin{aligned} x_1 &= A_1 e^{i\omega_1 t} + B_1 e^{-i\omega_1 t} \\ x_2 &= A_2 e^{i\omega_2 t} + B_2 e^{-i\omega_2 t} \end{aligned}$$

diagonalizzazione della matrice K/M

$$\det \begin{bmatrix} -\frac{k}{m_1} - \lambda & \frac{k}{m_1} \\ \frac{k}{m_2} & -\frac{k}{m_2} - \lambda \end{bmatrix}$$

$$\lambda^2 + \lambda(\frac{k}{m_1} + \frac{k}{m_2}) = 0 \implies$$

$$\lambda_1 = 0$$

$$\lambda_2 = -\frac{k}{\mu}$$

soluzione 1

$$\lambda = 0$$

$$\begin{bmatrix} -\frac{k}{m_1} & \frac{k}{m_1} \\ \frac{k}{m_2} & -\frac{k}{m_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$x = A \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

soluzione 2

$$\lambda = -\frac{k}{\mu} = \frac{k}{m_1} + \frac{k}{m_2}$$

$$\det \begin{bmatrix} -\frac{k}{m_1} + \frac{k}{m_1} + \frac{k}{m_2} & \frac{k}{m_1} \\ \frac{k}{m_2} & -\frac{k}{m_2} + \frac{k}{m_1} + \frac{k}{m_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$x = A \begin{bmatrix} m_2 \\ -m_1 \end{bmatrix}$$